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The range–energy relation for 0·1–600 keV electrons

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Abstract. Available data on the range of electrons with energies less than 600 keV in air and other light materials are reviewed critically. It is shown that the practical (extrapolated) range R_p (in g cm^{-2}) can be related to the energy E (in keV) by using the following expressions:

$$x = \lg E; \qquad y = \lg R_p$$

$$y = -6{\cdot}160 + 2{\cdot}585x - 0{\cdot}220x^2 \qquad (E > 100 \text{ keV})$$

$$y = -5{\cdot}100 + 1{\cdot}358x + 0{\cdot}215x^2 - 0{\cdot}043x^3 \qquad (0{\cdot}1 < E < 100 \text{ keV}).$$

The need for further work at energies less than 10 keV is stressed.

1. Introduction

A knowledge of the relation between the initial energy of electrons and their range in matter is important in the theory of the effects produced in the upper atmosphere by electrons precipitated into it and is also of interest in certain branches of solid state physics, biophysics and nuclear physics. The subject has most often been reviewed from the latter point of view (Flammersfeld 1947, Glocker 1948, Katz and Penfold 1952, Birkhoff 1958), in which emphasis is laid on energies above 100 keV, which are most appropriate to beta ray studies. Some reviews have been published by upper atmosphere physicists (Chamberlain 1961, Dalgarno 1961, 1962), but most workers appear to have assumed that one of the formulae proposed for the higher energy region can be extrapolated to lower energies without much error. Thus Rees (1963), in one of the most frequently-quoted papers on the subject, used the expression

$$R = 4{\cdot}57 \times 10^{-3} E^{1{\cdot}75} \text{ mg cm}^{-2} \qquad (E \text{ in keV}), \tag{1}$$

which describes the experimental work of Grün (1957) over the energy interval 5–54 keV, and assumed that it could be used down to 0·4 keV. Later, however, he published (Rees 1964) a graph showing experimental points due to Alper (1932), theoretical ones due to Dalgarno (1961) and some taken from Chamberlain (1961), which suggest that the range is in fact greater than that predicted by equation (1) in the region below 2 keV. Gledhill and van Rooyen (1963) took the formula due to Katz and Penfold (1952):

$$R = 412 E^{1{\cdot}265 - 0{\cdot}954 \ln E} \text{ mg cm}^{-2} \qquad (E \text{ in MeV}) \tag{2}$$

and assumed it to be valid below the lower limit of 10 keV at which those authors had fitted it to experimental data. In fact, it is quite inaccurate in this region as figure 1

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shows. Berger *et al* (1970) have given a table of values of the practical range R_p calculated theoretically for various energies in the interval 2–20 keV, and shown to agree well with the experimental results of Grün (1957) between 5 and 20 keV. Wulff and Gledhill (1973) have fitted the formula:

$$R_p = 4{\cdot}17 \times 10^{-6} E^{1{\cdot}74} \text{ g cm}^{-2} \qquad (E \text{ in keV}) \tag{3}$$

to the figures found by Berger *et al* (1970) for the case where the electron flux is isotropic over the downward hemisphere, and have assumed that this expression is valid down to 1 keV, though they pointed out that the matter is worthy of further investigation.

In the present paper a survey is made of available experimental and theoretical data on the range and energy in the region below 600 keV. Special emphasis is laid on energies below 10 keV, which are of great importance in the study of the effects produced by electrons precipitated into the atmosphere.

2. Experimental range–energy data

Before discussing the available experimental values it is desirable to comment on the meaning of the term 'range'. Curves showing the dependence of, for example, transmitted electron flux on absorber thickness or incident electron energy, tend to approach the zero level (or background level) asymptotically, due to the statistical nature of the processes leading to energy loss. Such curves almost always show, however, a straight portion which can be readily identified and extrapolated to the zero level to give the 'practical range' R_p. This concept was used as long ago as 1915 by Varder (1915). Flammersfeld (1947) and Katz and Penfold (1952) have discussed in detail the extrapolation of the range, especially if a beta emitter is used as the source of electrons, which are then not mono-energetic unless magnetic separation is used. Fortunately the extrapolated range at the end-point of the spectrum R_β appears to depend on the end-point energy in exactly the same way as R_p does on the energy of mono-energetic electrons (Katz and Penfold 1952). Other concepts such as the 'true range' and the 'maximum range' are discussed by the above authors.

Several experimentalists have reported ranges which approximate much more closely to the maximum range than to the practical range. Where it is clear that no extrapolation has been carried out and that the range given is the maximum one, a correction has been made as follows to give the practical range: from the theoretical values for the maximum range in air R_0 given by Berger *et al* (1970) in their table 2 and from the values of the practical range R_p at the same energies given in their table 6, it is possible to show that the ratio R_p/R_0 varies approximately linearly with energy from 0·82 at 2 keV to 0·87 at 20 keV. Values of R_0 have therefore been converted to R_p by multiplication by the appropriate value of this ratio at the energy concerned.

The range also differs in cases where the incident electron beam is mono-directional from those in which it is not so. This is illustrated in table 5 of Berger *et al* (1970), in which the range for the monodirectional case is always about 8 % greater than that for the IDH (isotropic over the downward hemisphere) case. Since practically all the experimental data have been obtained with a monodirectional beam of electrons incident on the absorbing material, we shall discuss only this case initially and consider later its relation to the other case.

There is both theoretical (Bethe 1933, Spencer 1955) and experimental (Holliday and Sternglass 1959) evidence that the range of electrons in a material depends on the

atomic number, though this dependence is very weak at energies of a few keV and less. In the present analysis, values for the range in materials containing atoms up to aluminium ($Z = 13$) have been accepted, but the large amount of data on ranges in substances such as heavy metals and nuclear emulsions has not been considered at all.

The smallest ranges appearing in figure 1 were measured by Anslow (1925). She accelerated electrons from a hot filament and allowed them to enter an air-filled ionization chamber through a metal capillary tube. The ionization chamber was hemispherical in form, with the end of the capillary at its centre. The ionization current was measured as a function of pressure in the chamber. For a given energy, a critical pressure was found, above which the ionization current was constant. This was interpreted to mean that the electron beam was now totally absorbed in the gas in the chamber, without striking the wall, and hence that the range at the critical pressure was equal to the radius of the chamber. No mention is made of any extrapolation to find the practical range. The measured values, read off carefully from Anslow's figure 4, lie well on a straight line when converted to g cm^{-2} and plotted on a log–log scale, as in figure 1. Anslow's results have been criticized by Lehmann and Osgood (1927), with

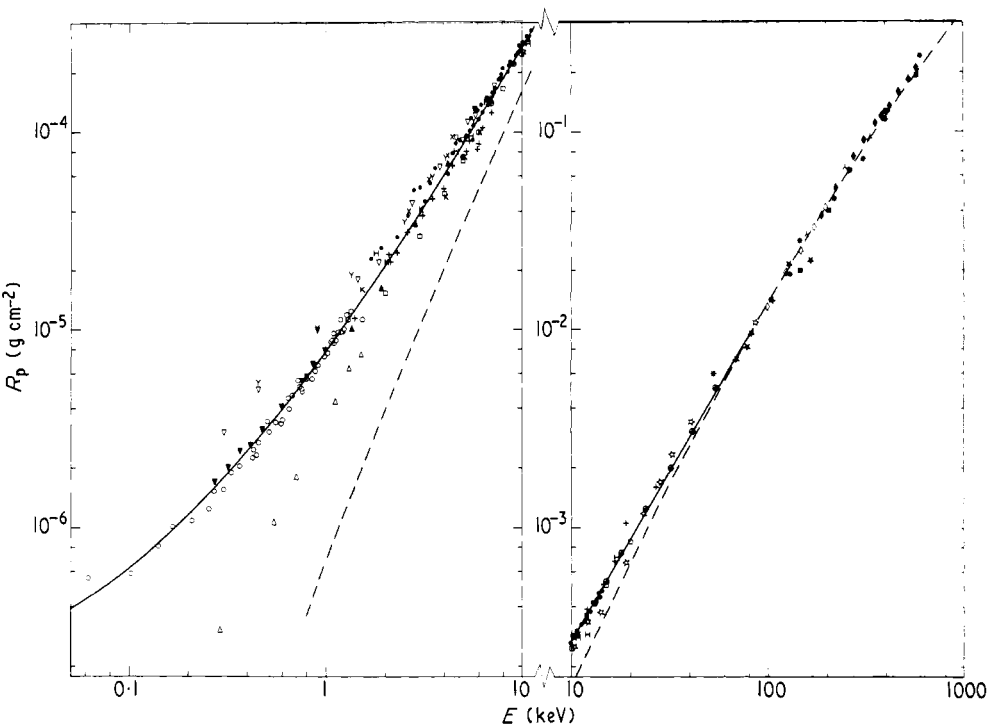

Figure 1. Available data on the practical range of mono-directional electrons in air, aluminium, aluminium oxide, collodion and formvar for energies below 600 keV. The broken line is from the Katz–Penfold formula; the full curve is from equation (5). Experimental points are as follows: △ Anslow (1925), ○ Hartman (1968), ▼ Lehmann and Osgood (1927), ▽ Young (1958), Y Young (1956), ▲ Alper (1932), + Lane and Zaffarano (1954), □ Berger *et al* (1970), K Klemperer *et al* (1960), ● Butkevich and Butslov (1958), H Holliday and Sternglass (1959), X Hoffman (1955), ◔ Grün (1957), ☆ Schonland (1925), * Glendenin (1948), ★ Gentner (1931), ◇ Carlvik (1953), ⊹ Varder (1915), ■ Eddy (1929), ◌ Madgwick (1927), ⅄ Seliger (1955), ◆ Marshall and Ward (1937).

whose values they disagree by a factor of about 10 at 150 eV, on the grounds that she took no precautions to ensure that her electron beam was homogeneous in energy. The effect of inhomogeneity in the incident beam would be to make the end point of the curve more difficult to estimate and so to stress the need for some kind of extrapolation; when carried out this would almost certainly *decrease* the final values even below the ones reported by Anslow. A more serious criticism may be made, however. Anslow used a McLeod gauge connected very close to the ionization chamber for her pressure measurements, apparently without any cold trap. If mercury vapour had diffused into the ionization chamber it would increase markedly the stopping power of the gas without producing a corresponding effect on the measured values of the pressure, from which the range was calculated. Thus the measured range would be too low, especially at the smallest pressures. For this reason, although they are shown in figure 1 to stress the discordance among observed values at low energies, Anslow's data have not been used in the determination of the relation between range and energy given later.

Hartman (1968) has published data covering the energy interval 0·06–1·32 keV. He obtained his results in the course of an investigation of the efficiency of excitation of various spectral features in the luminescence produced by electrons in air. A beam of electrons of known energy entered a large photometric integrating sphere containing air at low pressure and the resulting light output, in the selected spectral region, was recorded as a function of pressure. The light intensity was found to increase with increasing pressure until it reached a 'break-point', at which the slope of the graph decreased sharply and approached a 'saturation' value which, however, was not constant, but fell slowly with increasing pressure. The method is thus, in many ways, an optical analogue of Anslow's experiment. The break point was initially identified as the pressure at which the range was equal to the diameter of the sphere, but Hartman found that a second point at a higher pressure, at which the curve became linear in the saturation region, agreed better with other measurements and he therefore adopted this point to determine the range. From his figure 11 it appears that this choice may introduce an uncertainty as large as 20 μm into the value given for the pressure which corresponds to the range, and this in turn an uncertainty which could be as large as 50 % in the range itself. Therefore the points measured by this technique have been ignored and are not shown in figure 1. In a second method, however, Hartman measured the glow with the aid of a photometer having a narrow field of view, placed at a known distance from the electron gun. Curves of light output against energy of incident electrons are shown in Hartman's figure 13, with linear extrapolations corresponding closely to the usual determination of R_p. An allied technique used a movable optical probe with a light guide to explore the light emission at various distances from the electron source. Values obtained by these two methods, which are clearly much more reliable than the first one, are shown in Hartman's figure 12; they have been carefully read off from it and are plotted in figure 1.

Measurements by a very unusual technique have been reported by Davis (1954, 1955). Freeze dried enzymes on glass cover slips were bombarded with electrons of known energy, in the range 0·1–2·0 keV, and then assayed chemically or biologically. Assuming the value 1·3 g cm^{-3} for the density of the dried enzyme, the range can be found. The values so calculated are even lower than those reported by Anslow (1925), which they support, however, against those of Hartman and of Lehmann and Osgood. Careful consideration suggests that the accuracy of this method is very questionable, however, and the results have therefore been discarded and do not appear in figure 1.

Lehmann and Osgood (1927) carried out a series of experiments very similar in

principle to those of Anslow (1925). They took special care to make their electron beam as homogeneous as possible and they used a trap between their McLeod gauge and the ionization chamber. On reading their paper it appears that the ranges which they report, corresponding to the critical pressures at which the number of ions per electron became constant, are maximum ranges, but in the following paper in the journal Lehmann (1927) states that 'the critical pressure, just adequate to absorb the electrons, may be determined by extrapolating to their point of intersection the linear portions of the curves'. Comparison of values for the range in air read off from the corresponding graph in the latter paper confirms that the points plotted in figure 7 of Lehmann and Osgood's paper are in fact practical ranges. They therefore appear unaltered in figure 1 of the present paper.

Other measurements below 1 keV to be seen in figure 1 are due to Young (1958). He prepared films of aluminium oxide with thicknesses between 85 and 5000 Å by anodizing aluminium and then etching the metal substrate away, a technique first used by Hoffman (1955). The thickness was found from an empirical relationship between thickness and anodizing voltage, established by weighing larger sheets. Practical ranges were found by extrapolating curves showing the fraction of electron current transmitted by the film against initial energy. Space charge effects were shown to be absent by increasing and decreasing the beam current by a factor of 100, when no change in the ratio was observed. Values taken from Young's table are plotted in figure 1. They lie on the high side of the other results. The Bethe (1933) theory shows that the stopping power depends on the atomic number and the effective ionization energy of the material, but only through a logarithmic term. Calculation shows that this effect would indeed decrease the range in air below that in Al_2O_3, but only by a very small fraction of the discrepancy in figure 1. Becker (1927, 1929), Katz (1938) and Butkevich and Butslov (1958) have commented on the presence of 'holes' in thin metal foils. If such holes were present in Young's films, this would have led to an overestimate of the thickness required to absorb the electrons and thus give values of the practical range which were too large, as observed. The same criticism has been made (Holliday and Sternglass 1959) of Young's (1956) determination of the range by evaporating thin films of aluminium on to ZnS phosphor layers bound by collodion, removing the collodion by heating and then detecting penetration by the electron beam by observing the onset of luminescence in the phosphor. The good agreement between the results of these two rather different methods, shown in figure 1, leads one to believe that there may perhaps be a real difference between the stopping power of aluminium oxide and air at these low energies, since it seems unlikely that precisely similar holes would appear in films prepared in such different ways. In view of these doubts, Young's results, although shown in figure 1 to emphasize the discrepancies at low energies, have been ignored in fitting a formula to the experimental points.

Alper's (1932) measurements were made by studying the lengths of delta ray tracks in stereoscopic cloud chamber photographs. Accepting the theoretical result that the maximum possible speed of the delta ray electron is twice that of the incident alpha particle, Alper was able to relate the maximum length of track to the corresponding speed and from this the range–energy relationship is easily obtained. The ranges found in this way are the maximum ones and the values taken from Alper's table have been converted to practical ones, by the method explained earlier, before plotting in figure 1.

Lane and Zaffarano (1954) studied the transmission of electron beams by thin films of formvar, collodion and aluminium and determined the practical ranges in these

substances by extrapolation. Points taken from figure 3 of their paper are shown in figure 1. They agree well with the values given by other workers using different techniques.

Klemperer *et al* (1960) used films of aluminium evaporated on to rock salt, which was later dissolved away. They also used aluminium oxide films prepared by an electrolytic process like that employed by Young (1958). The films were examined under an electron microscope and found to be 'grainless'. Values of the extrapolated range R_p taken from their table (p 712 of their paper) are found to agree well with those of other workers at similar energies and are shown in figure 1.

Butkevich and Butslov (1958) investigated the emission of secondary electrons from the far side of aluminium foils through which electrons in the energy interval 1·7–14·3 keV were passing. They give a small graph, their figure 5, showing the resulting range–energy relation. The numerous experimental points shown on this diagram have been read off as accurately as possible. Since Butkevich and Butslov defined the range as the thickness at which the ratio of secondary to primary electron current reaches 1%, these are close to maximum ranges and they have been converted to practical ranges, as described earlier in this paper, before plotting in figure 1.

In the experiments of Holliday and Sternglass (1959) thin films of aluminium were evaporated on to gold substrates and the change in back-scattered electron intensity, which occurred when the electrons penetrated the aluminium layer, were measured; R_p was found by extrapolation. The points shown in figure 1 were read off from figure 5 of their paper.

Hoffman (1955) studied the transmission of electrons with energies between 2·65 and 4·40 keV through aluminium oxide films from 1000 to 3000 Å thick, prepared by the electrolytic technique later adopted by Young (1958) and by Klemperer *et al* (1960). He evaporated thin layers of aluminium on to the surface of his films to avoid space charge effects. His four points for the extrapolated range R_p are shown in figure 1. Like those of Young at lower energies, they lie on the high side of the graph.

Grün's (1957) work has been a popular reference with upper atmosphere physicists. He studied the spatial distribution of luminescence excited in air by incident electron beams with energies from 5 to 54 keV. His illustration of the isophotes for 32 keV electrons has been used by Rees (1963), Stolarski (1968) and others in the study of the effects of angular distributions of incident electrons other than monodirectional ones. The points plotted in figure 1 were read off from Grün's graph of the extrapolated ranges R_p.

Among the higher energy results those of Carlvik (1953) call for the comment that they appeared first in a footnote to the review by Katz and Penfold (1952), but have since appeared in the literature as cited, with a very detailed description of the experimental techniques used. Carlvik records both the maximum range R_0 and the practical range R_p. It is surprising that the ratio R_p/R_0 varies from 0·55 to 0·6 as the energy goes from 100 to 200 keV. This ratio is much smaller than that found by Berger *et al* (1970) theoretically for air in the range 2–20 keV and this suggests that it would be unwise to correct values of R_0 at energies above 20 keV by multiplying by a factor to get R_p, until this discrepancy has been thoroughly studied.

The data due to Schonland (1923, 1925), Varder (1915), Glendenin (1948), Eddy (1929), Madgwick (1927) and Marshall and Ward (1937) have been reviewed in the works cited in the introduction to this paper. The work of Gentner (1931) seems to have been overlooked by most other reviewers. The four points from his table 1 have been converted to the appropriate units (he had already extrapolated to find R_p) and

are plotted in figure 1. Those due to Seliger (1955) are more recent and have been extrapolated from figure 3 of his paper, using the curves for electrons.

The work of Gubernator (1958) was very carefully conceived and executed. He separated beta rays, both positrons and electrons, electromagnetically to get mono-energetic beams, and also used thermionic electrons at lower energies. His results cover the energy interval 10–160 keV and are shown in figure 2 of his paper, where 50 points are plotted for electrons (and 33 for positrons). Unfortunately the range reported is the maximum range and although conversion of the values in the range 10–20 keV, by the technique discussed earlier, gives points which lie in the middle of the densely populated band in figure 1, the doubt about the value of the correcting factor, discussed above under the work of Carlvik (1953), has led to their exclusion from the set of data accepted for curve-fitting. They are therefore not shown in figure 1, where they would merely clutter a portion of the diagram which is already very congested. It is interesting to note that if Gubernator's results at energies above 20 keV are corrected on the basis that $R_p/R_0 = 0{\cdot}87$, they agree excellently with those of others. This suggests that Carlvik's value for this ratio is too low.

Das Gupta and Ghosh (1946) have given a table of values of the range for various energies which has often been quoted as a source of data. Unfortunately they do not indicate the authors of the figures which they give and these have consequently not been included in the present evaluation. Khare and Varshni (1961) have also given a table of values and references, which have been carefully compared with those accepted in the present paper. They have also discussed various formulae relating the range and the energy in the lower energy region.

3. Theoretical range–energy data

Most theoretical work has relied on the Bethe (1933) formula for the stopping power of a substance for electrons. The theory has been reviewed in some detail by Dalgarno (1962) and tables of values have been given, *inter alia* by Heitler (1954), Spencer (1955), Chamberlain (1961) and Dalgarno (1961). Berger *et al* have made a thorough theoretical examination of the problem in the energy interval 2–20 keV and have included the effects of straggling. They have extrapolated their data to give values of R_p, which should thus be entirely compatible with the experimental points given in figure 1. Their nine theoretical points have therefore been included with the experimental ones, though they tend to lie on the low range side of the graph and show a tendency to deviate more at the lowest energies.

At higher energies the calculations of Spencer (1955) are the most extensive. Unfortunately straggling was neglected in this work and a conversion to R_p like that based upon the work of Berger *et al* at lower energies is not feasible. Spencer's data have therefore reluctantly been omitted from further consideration.

4. Discussion

The agreement between values obtained by a variety of techniques appears at first to be encouraging, as figure 1 shows. However, log–log plots of this type give rather a false impression of the consistency of data and some individual values in fact deviate from the mean curve by more than 25%, neglecting the points due to Anslow (1925)

and Young (1956, 1958). Clearly there is an urgent need for more measurements and theoretical values below 10 keV, and especially below 2 keV where the good agreement is between the results of two laboratories only.

It is evident that the graph has a point of inflexion in the vicinity of 40 keV. The broken line in figure 1 was calculated from the expression due to Katz and Penfold (1952), equation (2) above, after conversion of the units. It fits reasonably well above 100 keV but it has the wrong curvature to fit at lower energies.

Attempts to fit a third degree polynomial to the whole set of data from 0·06 to 600 keV did not give very satisfactory results and it seems simpler to accept the Katz–Penfold formula for energies above 100 keV and to fit a new expression below this. When converted to our units, and writing

$$x = \lg E \qquad \text{and} \qquad y = \lg R_p \tag{4}$$

the relation due to Katz and Penfold reads

$$y = -6{\cdot}160 + 2{\cdot}585x - 0{\cdot}220x^2 \qquad (E > 100\ \text{keV}). \tag{5}$$

Below 100 keV the continuous line in figure 1 represents the expression

$$y = -5{\cdot}100 + 1{\cdot}358x + 0{\cdot}215x^2 - 0{\cdot}043x^3 \qquad (0.1 < E < 100\ \text{keV}). \tag{6}$$

This gives a very satisfactory fit to the accepted data and joins smoothly on to the Katz–Penfold formula at 100 keV. Equations (5) and (6) may thus be used in conjunction to give a ready and rapid method of computing the range in air, as accurately as it is at present known, for energies of interest in the theory of the effects of electrons precipitated into the upper atmosphere. They may also be used, with somewhat less assurance, to compute the range in light solids such as aluminium; in this case it is possible that the range will be underestimated below about 5 keV.

5. Isotropic electron flux

Finally, we may note that the practical range for the case where the electron flux is isotropic over the downward hemisphere can be estimated by making use of the ratio $R_{p(IDH)}/R_{p(MD)}$ (MD = mono-directional), found from the entries in table 5 of the paper by Berger *et al* (1970). The ratio has the mean value of 0·926, individual values differing from this by less than 0·5 % over the energy interval 2–20 keV. If we make the questionable assumption that this ratio remains constant over our extended energy interval, then we may estimate $\lg R_{p(IDH)}$ by subtracting 0·033 from each of the above expressions, so obtaining

$$\lg R_{p(IDH)} = -6{\cdot}193 + 2{\cdot}585x - 0{\cdot}220x^2 \qquad (E > 100\ \text{keV}) \tag{7}$$

and

$$\lg R_{p(IDH)} = -5{\cdot}133 + 1{\cdot}358x + 0{\cdot}215x^2 - 0{\cdot}043x^3 \qquad (0{\cdot}1 < E < 100\ \text{keV}). \tag{8}$$

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